Indian J. Phys. **43**, 385—390 (1969)

Acoustic amplification in semi-conductors in the presence of external electric and magnetic fields

By A. Singh

Radio and Electronics Research Laboratory, Institute of Technology, Banaras Hindu University, Varanasi-5., U.P.

(*Received* 1 *January* 1969, *Revised* 24 *April* 1969)

The dispersion equation is derived, on the basis of the equation of stress, taking into account the electron sound wave interaction, and the amplification of the acoustic wave, due to its interaction with the conduction electrons in the semiconductors, both piezoelectric and non-piezoelectric, in the presence of the external fields, is investigated It is found that the amplification of the acoustic wave occurs only when the carrier drift velocity in the direction of propagation exceeds the velocity of sound. The geometric and cyclotron resonances are also found under the condition of amplification.

Introduction

A number of investigations (Hutson *et al* 1961, Pippard 1963, Solymar 1964 1966, 1967, Singh 1968, 1969 and Spector 1968) have been made of the amplification of the acoustic wave due to its interaction with the charge carriers in the semiconductors. However, there is a general agreement on the fact that the interaction becomes very much pronounced and the amplification occurs when the drift velocity, imparted to the conduction electrons in the direction of propagation of the acoustic wave by the dc fields, exceeds the velocity of sound. In this paper we undertake the study of interaction of charge carriers in the semiconductors in the presence of external dc fields, with the acoustic wave by deriving a dispersion equation for a coupled electron stream and the acoustic wave.

Dispersion Equation

We consider a longitudinal acoustic wave propagating in the x-direction of the medium and define a strain S and a stress T such that

$$S = \frac{\partial u}{\partial x} \tag{1}$$

and

$$\frac{\partial T}{\partial x} = \rho_m \frac{\partial^2 u}{\partial t^2} \tag{2}$$

where ρ_m^* is the mass density and u the local displacement of material in the direction of propagation of the acoustic wave. The stress tensor T can be determined from the internal energy of the medium and for an ordinary solid is given by $C.S$ where C is the elastic stiffness.

We assume that the acoustic wave interacts with conduction electrons through the deformation of the energy bands :

$$E_i = V_i S \tag{3}$$

where E_i is the interaction energy and V_i the deformation potential for the carriers of type i. In case of piezoelectric semiconductor there is an additional contribution to the internal energy from the polarisation fields which accompany the lattice displacement. Therefore the equation of stress in the presence of electron-sound interaction (Mason 1950, Weinreich 1956) is

$$T = CS - nV - dE \tag{4}$$

where d is the piezoelectric constant, n the electron density, ϵ the dielectric constant and

$$E = \text{Electric field due to free charges and polarisation}$$
$$= E_1 - (d/\epsilon) S. \tag{5}$$

The equation of continuity and Poisson's equation are

$$e \frac{\partial n}{\partial t} + \frac{\partial J}{\partial x} = 0 \tag{6}$$

$$\partial E_1/dx = (ne/\epsilon) \tag{7}$$

The conductivity σ for the free electron gas (Carleton & Auer 1965) is

$$J = \sigma [E + iqeVS] \tag{8}$$

We assume that all the phyiscal quantities can be regarded as the superposition of an unperturbed term and perturbed term, the latter varying as exp $[i(qx - \omega t)]$. Using equation (4) together with equations (5)-(8) and neglecting the product of V and d we obtain the following dispersion equation (for details see the appendix).

$$\omega^2 - q^2 V_{s_0}^2 = q^2 V_{s_0}^2 (K_1 + K_2) [(\sigma/\epsilon)/(i\omega - \sigma\epsilon)] \tag{9}$$

where q is sound wave number, V_{s_0} the velocity of sound in absence of perturbation and $K_1 = \frac{d^2}{C\epsilon}$, $K_2 = q^2 V^2 \epsilon/\rho_m V_{s_0}^2$.

The RHS term in equation (9) represents the perturbation, in q, assumed to be small. We re-write therefore equation (9) as

$$q = \frac{\omega}{V_{s_0}} + \frac{1}{2} \cdot \frac{\omega}{V_{s_0}} \cdot (K_1 + K_2) \left(\frac{\sigma/\epsilon}{-i\omega + \sigma/\epsilon} \right) \qquad \ldots(10)$$

Semiconductor in dc electric field.

Application of the dc electric field imparts a net drift velocity (V_d) to the conduction electrons in the direction of propogation of the

acoustic wave and if the drift velocity of the charge carriers exceeds the velocity of sound, amplification occurs. This can be readily seen from the following.

Case I : $ql<1$ where l is the electron mean free path.

Under this condition σ has the form (Spector 1962)

$$\sigma = \sigma_0/[\ \mu + iql\ V_F/3\ V_{s_0}\ ] \qquad ...(11)$$

where σ_0 is the dc conductivity, V_F the Fermi velocity and $\mu = (1 - \phi)$ where $\phi = V_d/V_{s_0}$. Substituting equation (11) in equation (10) and considering only the imaginary part of q, because the real part gives the acoustic velocity in semiconductor (Singh 1969), we have the following expression for the absorption coefficient.

$$\alpha(\phi) = \frac{K}{2} \cdot \frac{\omega_p^2 \tau}{V_{s_0}} \left[\frac{\mu}{\mu^2 + \left(\frac{\omega_p^2 \tau}{\omega}\right)^2 \left[1 + \frac{1}{3}\ (\omega/\omega p)^2 . (V_F/V_{s_0})^2 \right]^2} \right] \qquad ...(12)$$

where $K = (K_1 + K_2)$ and ω_p is the plasma frequency. The equation (12) agrees with that obtained by Weinreich (1956) and Spector (1962) for non-piezoelectric semiconductor. It also agrees with the result derived by Hutson *et al* (1961) for piezoelectric semiconductor provided $(\omega\ V_F/\omega p\ V_{s_0})$ is negligibly small. There is a cross over from absorption to amplification when the drift velocity exceeds the velocity of sound. The maximum in the absorption coefficient occurs at

$$\phi = 1 \pm (\omega_p^2 \tau/\omega) \left[1 + \frac{1}{3} \left(\omega V_F/\omega_p V_{s_0} \right)^2 \right]. \qquad ...(13)$$

The + ve sign corresponds to the amplification and − ve sign to the absorption.

Case II.

When $ql>1$, the conductivity σ becomes

$$\sigma = (3\sigma_0\ V_{s_0}/ql\ V_F)[(\pi\mu V_{s_0}/2V_F) - i] \qquad ...(14)$$

$$\text{therefore; } \alpha(\phi) = (\pi K \omega^3 V_F/12\omega_p^2 V_{s_0}^2)[\mu/\left[1 + \frac{1}{3}(\omega V_F/\omega_p V_{s_0})^2 \right]^2 \qquad ...(15)$$

which is in agreement with Spector's (1962) result. In this case the absorption coefficient increases linearly with drift velocity and there is no maximum in the absorption. The amplification occurs for drift velocities greater than the sound velocity.

SEMICONDUCTOR IN CROSSED ELECTRIC AND MAGNETIC FIELDS.

In the presence of crossed electric and magnetic fields (magnetic field being perpendicular to the direction of propagation) the carrier drift velocity is

$$\vec{V}_H = V_L (\vec{E} \times \vec{H})/H^2 \qquad ...(16)$$

where V_L is the velocity of light in the vacuum. The resonances in the absorption/amplification coefficient are obtained in the following two cases.

Case I : Geometric Resonance

Here the wave length (λ) of the acoustic wave is of the order of classical orbit radius. When $\omega_c \gg \omega$ and $\omega_c \tau \gg 1$ are satisfied the following expression σ is obtained (Spector 1963)

$$\sigma = \frac{3\sigma_0}{(ql)^2} \frac{(1-i\omega\tau Y)}{Y+(i/\omega\tau)} \frac{[1-g_0(X)]}{[1-g_0(X)]} \qquad ...(17)$$

where $g_0(X)$ is oscillatory function of X and $X = (q.R)$, R is orbital radius of an electron moving perpendicular to the magnetic field with Fermi velocity and is equal to (V_F/ω_C).

$$Y = (1 - V_H/V_{s_0}) \text{ and } \phi = V_H/V_{s_0}$$

Using equations (17) and (10) we obtain

$$\alpha(\phi) = \frac{3KV_{s_0}Y\tau}{2V_F^2} \cdot \frac{g_0(X)[1-g_0(X)]}{\omega^2\tau^2Y^2 + [1-g_0(X)]^2} \qquad ...(18)$$

which agrees with the result obtained by Spector (1963) for nonpiezoelectric semiconductor. The maxima in the absorption/amplification coefficient occur at

$$\phi = 1 \pm [1 - g_0(X)]/\omega\tau \qquad ...(19)$$

Case II : Cyclotron Resonance

In this case sound frequency is of the order of cyclotron frequency ω_c and the conductivity σ has the following form

$$\sigma = -\frac{3i\sigma_0 V_{s_0}}{qlV_F}\left[1 + \frac{i\omega\tau Y}{2ql} \coth(\pi/\omega_c\tau)(1 - i\omega\tau Y)\right] \qquad ...(20)$$

Therefore the absorption coefficient in this region is

$$\alpha(\phi) = \frac{\pi\omega KY}{4V_F} \cdot \frac{\tanh(\pi/\omega_c\tau)\sec^2(\pi\omega Y/\omega_c)}{\tanh^2(\pi/\omega_c\tau) + \tan^2(\pi\omega Y/\omega_c)} \qquad ...(21)$$

It is quite obvious from this equation that the amplification occurs when the carrier drift velocity exceeds the velocity of sound. We have

oscillations in the amplification/absorption coefficient as long as $\omega_c\tau>1$. The maxima in the absorption/amplification coefficient occur when

$$\omega Y = n\,\omega_c,\ (n = 1, 2, 3\ldots) \qquad \ldots(22)$$

This result agrees with that derived by Spector (1963) and Mikoshiba (1958) and also with that obtained by Cohen *et al* (1960) for drift velocity much less than the sound velocity *i. e.* when $Y = 1$.

Discussion.

Our calculations have shown that there is a dependence of the amplification of the acoustic wave on the carrier drift velocity and the amplification of the acoustic wave occurs when the carrier drift velocity exceeds the velocity of sound. This is because maximum interaction between the charge carriers and the acoustic wave occurs when the electrons have a net drift velocity in the direction of propagation greater than the velocity of the acoustic wave. The geometric resonance in absorption/amplification coefficient is associated with the Bessel function in the condictivity tensor. This has to do with the strength of the interaction between the particular orbit and the electric field rather than the resonant absorption/amplification of energy. In quantum mechanical language geometric resonance corresponds to the variation in the matrix element, rather than in the resonance denominators also appearing in the conductivity tensor. The cyclotron resonance corresponds to the variation of the resonance denominator. *i. e.* to the resonant absorption from or transfer of energy to the sound wave.

We have also found that the amplication of the acoustic wave in both piezoelectric and non-piezoelectric semiconductors increases with frequency as shown by equations (12), (13) and (18). It is therefore possible to amplify microwave acoustic waves.

The author feels very much indebted to Prof. S. S. Banerjee for his keen interest and several helpful comments on the manuscript.

Appendix.

We outline here briefly the derivation of the dispersion equation (9). From equation (4), using equations (2), (5) and (7) we get

$$\rho_m\omega^2 u = Cq^2 u + iqnV + \frac{d^2}{\epsilon} q^2 u + \frac{ne}{\epsilon} d \qquad \ldots(A1)$$

substituting J from equation (6) and E from (7) into equation (8) we obtain

$$\frac{ne\dot{\omega}}{q} = \sigma\left[-\frac{d}{\epsilon} iqu + \frac{ne}{iq\epsilon} - q^2 eVu\right] \qquad \ldots(A2)$$

Expressing n from equation (A2) in terms of u, substituting it into equation (A1) and assuming that $(dV) \ll 1$ and $K_1 \ll 1$, we obtain equation (9).

References

Carleton, H R. & Auer, P. L. 1965 *Solid St. Electron* 8, 285.

Cohen, M. H., Harrison, M. J. & Harrisson W. A. 1960 *Phys. Rev.* 117, 937.

Hutson, A. R., Mcfee, J. H. & White D. L. 1961 *Phys Rev. Lett.* 7, 237.

Mason, W. P. 1950 *Piezoelectric crystals and their applications to Ultrasonics*, Van. Nostrand, New York.

Mikoshiba, N. 1958 *J. Phys. Soc.* (Japan) 13, 759.

Pippard, A. B. 1963 *Phil. Mag.* 8, 161.

Singh, A. 1968 *Solid St. Electron* 11, 1097.

1969 *Int. J. Electron*, 25, 495.

1969 *Ind. Jr. Pure and Appl. Phys.* 7, 456.

Solymar, L. 1964 J. *Appl. Phys.* 35, 3420.

1966 *Solid. St. Electron* 9, 879.

1967 *Int. J. Electron* 22, 459.

Spector, H. N. 1962 *Phys. Rev.* 127, 1084.

1963 *Phys. Rev.* 131, 2512.

1968 *Phys. Rev.* 165, 562.

Weinreich, G. 1956 *Phys. Rev.* 104, 321.